# Relations FIRST and FOLLOW for Parsing Expression Grammar

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CS&P 2010

## What is Parsing Expression Grammar?

The last fad in top-down parsing with limited backtracking.

- 1961 Brooker & Morris Altas Compiler Compiler
- 1965 McClure TransMoGrifier (TMG)
- 1972 Aho & Ullman Top-Down Parsing Language (TDPL)
- ...
- 2004 Ford Parsing Expression Grammar (PEG)

```
number = real / integer
real = digits? "." digits
integer = digits
digits = [0-9][0-9]*
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Note: not LL(1).

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number

number->real

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digits = [0-9][0-9]*
29.165
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number->real->digits

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number->real->".": returns failure
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number->real: backtracks

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number->real: returns failure

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Once number succeeded, nothing can force it to try real. integer hides part of the language of real.



### PEG is not EBNF

#### All of these fail on input aab:

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("a"/"aa") "b" - "a" consumes a, "b" fails on ab

("aa"/"a") "ab"

("a"/"c"?) "aab"
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("aa"/"a")"ab"

("a"/"c"?)"aab"
```

Not easy to see what happens in a complex grammar.

$$A = "a"A"a" / "aa"$$

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aaaa	consumes	4	of	4
aaaaa		2	of	5
aaaaaa		4	of	6
aaaaaaa		6	of	7

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Result depends on input far ahead.

Programmer's paradise: write, try, debug, show your skill.



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- Suggestion: detect LL(1) violations.
- How: adapt known techniques to PEG.



## Classical FIRST and FOLLOW

A known technique to check for LL(1) uses these relations:

- FIRST(s) set of possible first letters in a string derived from grammar symbol s.
- FOLLOW(s) set of possible letters that can follow a string derived from grammar symbol s.

### FIRST and FOLLOW for PEG

#### Adapted to PEG:

- FIRST(e) set of terminals that may be invoked by expression e on the start of input.
- FOLLOW<sub>s</sub>(e) set of expressions that may be invoked after success of e.
- FOLLOW<sub>f</sub>(e) set of expressions that may be invoked after failure of e.

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#### Example:

```
e_1 = "abc" [a-z]^* FIRST(e_1) = {"abc"}, e_2 = "abd" [a-z]^* FIRST(e_2) = {"abd"}, e_3 = [a-z] [a-z]^* FIRST(e_3) = {[a-z]}
```

 $e_1$  and  $e_2$  are disjoint.  $e_2$  and  $e_3$  are not.

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- Language hiding does not occur in a disjoint choice.
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- ② If any of  $e_1, \ldots, e_n$  in a disjoint choice fails after succeeding with at least one terminal, no terminal will succeed on that input. (Until the parser backtracks and takes another try.)
  - We can stop trying other alternatives. This a PEG version of predictive parsing. (Mizushima, Meada & Yamaguchi)



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But this is a long story... See CS&P 2008, Fundamenta Inf. 93.

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- ⊕ Good news: experiment with a large grammar (Java 1.6) found 264 of 329 choice and star expressions to be disjoint.
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Let us see why.

Lookahead expression: !e where e is any expression.

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- Make sure the input does not start with abc.
- But do not consume anything.

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#### In other words:

- Make sure the input does not start with abc.
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- "abc" is included in FIRST.

#### Consider

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 $e_1$  consumes strings of letters that do not start with  ${\tt abc}.$ 

e<sub>2</sub> consumes strings of letters that do start with abc.

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\label{eq:first} \begin{split} \mathsf{FIRST}(e_1) &= \{\texttt{"abc", [a-z]}\} \; (\mathsf{yes}, \, e_1 \; \mathsf{tries \; both}), \\ \mathsf{FIRST}(e_2) &= \{\texttt{"abc"}\}. \end{split}
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They are flagged as non-disjoint.

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We need something like  $FIRST(e_1) = \{ [a-z] \text{ but not "abc"} \}.$ 

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Only [a-z] is called to really bite off a piece of input, while "abc" is trying to prevent this.

Leaving "abc" out does not help:  $FIRST(e_1) = \{ [a-z] \} \text{ and } FIRST(e_2) = \{ "abc" \}$ are still not disjoint.

We need something like  $FIRST(e_1) = \{[a-z] \text{ but not "abc"}\}$ . Unfortunately, this does not work in general. We need something new.

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 $[a-z]^*$  bites any string in  $[a-z]\Sigma^*$ .

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### Example:

```
[a-z]^* bites any string in [a-z]\Sigma^*. "abc" [a-z]^* bites any string in "abc"\Sigma^*. (!"abc")[a-z]^* bites any string in \overline{"abc"}\overline{\Sigma^*}\cap [a-z]\Sigma^*.
```

Define BITES(e) as a set of strings that e may bite: e bites  $s \Rightarrow s \in BITES(e)$ .

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$$e ext{ bites } s \Rightarrow s \in ext{BITES}(e).$$

Examples:

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$$\mathsf{BITES}\big( \texttt{(!"abc")} \, [\mathsf{a-z}]^* \big) = \overline{\texttt{"abc"} \Sigma^*} \cap [\mathsf{a-z}] \, \Sigma^*.$$

# New disjointness

 $\mathsf{BITES}(e_1) \cap \mathsf{BITES}(e_2) = \varnothing$  means:

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 $\mathsf{BITES}(e_1) \cap \mathsf{BITES}(e_2) = \varnothing.$ 

"abc"  $[a-z]^*$  and  $(!"abc")[a-z]^*$  are now disjoint!

# Updated main results

Redefine " $e_1/\dots/e_n$  disjoint" to mean

" $e_1, \ldots, e_n$  are pairwise disjoint in the new sense."

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Redefine " $e_1/\dots/e_n$  disjoint" to mean " $e_1,\dots,e_n$  are pairwise disjoint in the new sense."

- Language hiding does not occur in a disjoint choice.
- If any of e<sub>1</sub>,..., e<sub>n</sub> in a disjoint choice fails after biting the input, nothing will bite that input. (Until the parser backtracks and takes another try.)

# Everything fine? Not really...

The lookahead is still a problem.

$$\mathsf{BITES}((!e_1)e_2) = \overline{\mathsf{SUCC}(e_1)} \ \cap \ \mathsf{BITES}(e_2)$$

where  $SUCC(e_1)$  should be the set of strings on which  $e_1$  succeeds.

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where  $SUCC(e_1)$  should be the set of strings on which  $e_1$  succeeds.

Finding SUCC(e) for arbitrary e is difficult.

It is about *e* succeeding on *s*, not just biting it. And remember, it may depend on input far ahead. (Back to square one?)

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Otherwise we can approximate SUCC "from below",

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$$\widetilde{\mathsf{SUCC}}(e) \subseteq \mathsf{SUCC}(e)$$
.

(We have to preserve "e bites  $s \Rightarrow s \in \mathsf{BITES}(s)$ ".)

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(We have to preserve "e bites  $s \Rightarrow s \in \mathsf{BITES}(s)$ ".)

One such approximation is  $\widetilde{\mathsf{SUCC}}(e) = \varnothing$ 

which gives  $BITES((!e_1)e_2) = BITES(e_2)$ , loosing all info on  $e_1$ .

Not good, but I do not see any better yet.

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Of course, the emptiness problem for such expressions is decidable, but standard procedures are cumbersome with a large alphabet.

### Conclusions

- BITES is better than FIRST, but still not perfect.
- BITES is more difficult to implement, but this is one-off, not run-time, analysis.
- There is still much left to be detected.

### What next

- Implement and see how it works?
- Porget it?
- More research? (Need something for CSP 2011...)

That's all folks ...

Thanks for your attention!